University of California, Irvine

CS 277: Control and Reinforcement Learning

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Review

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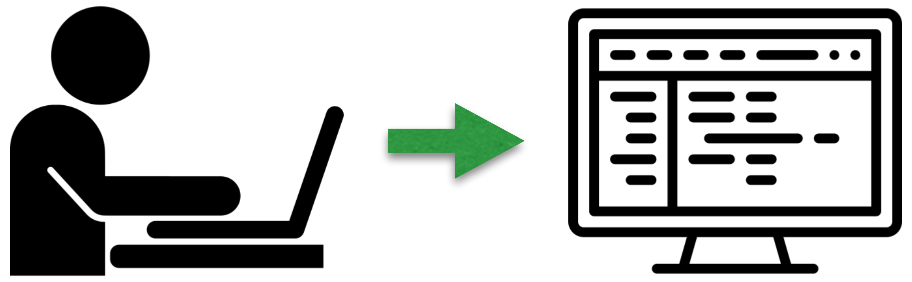
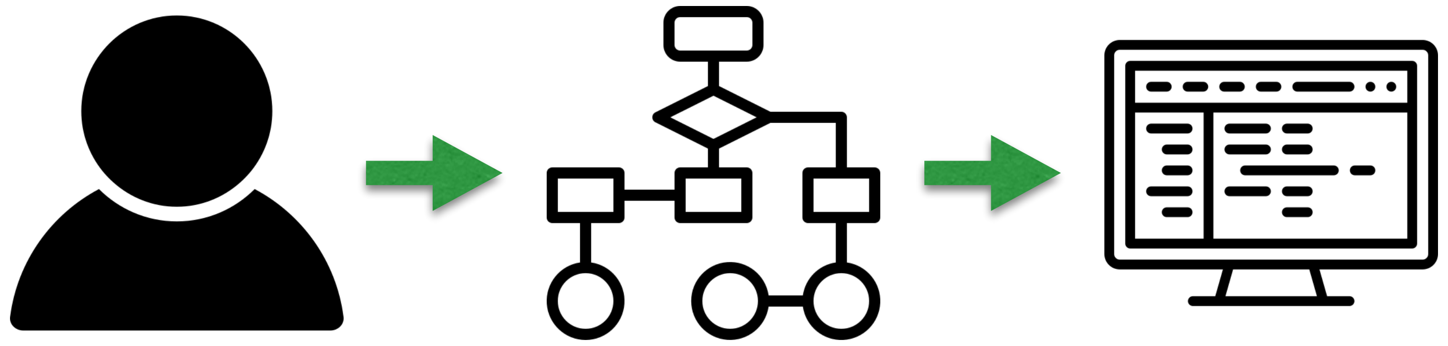
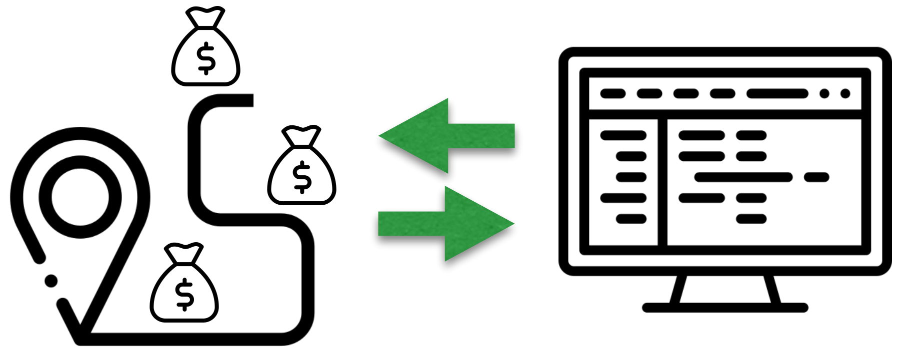
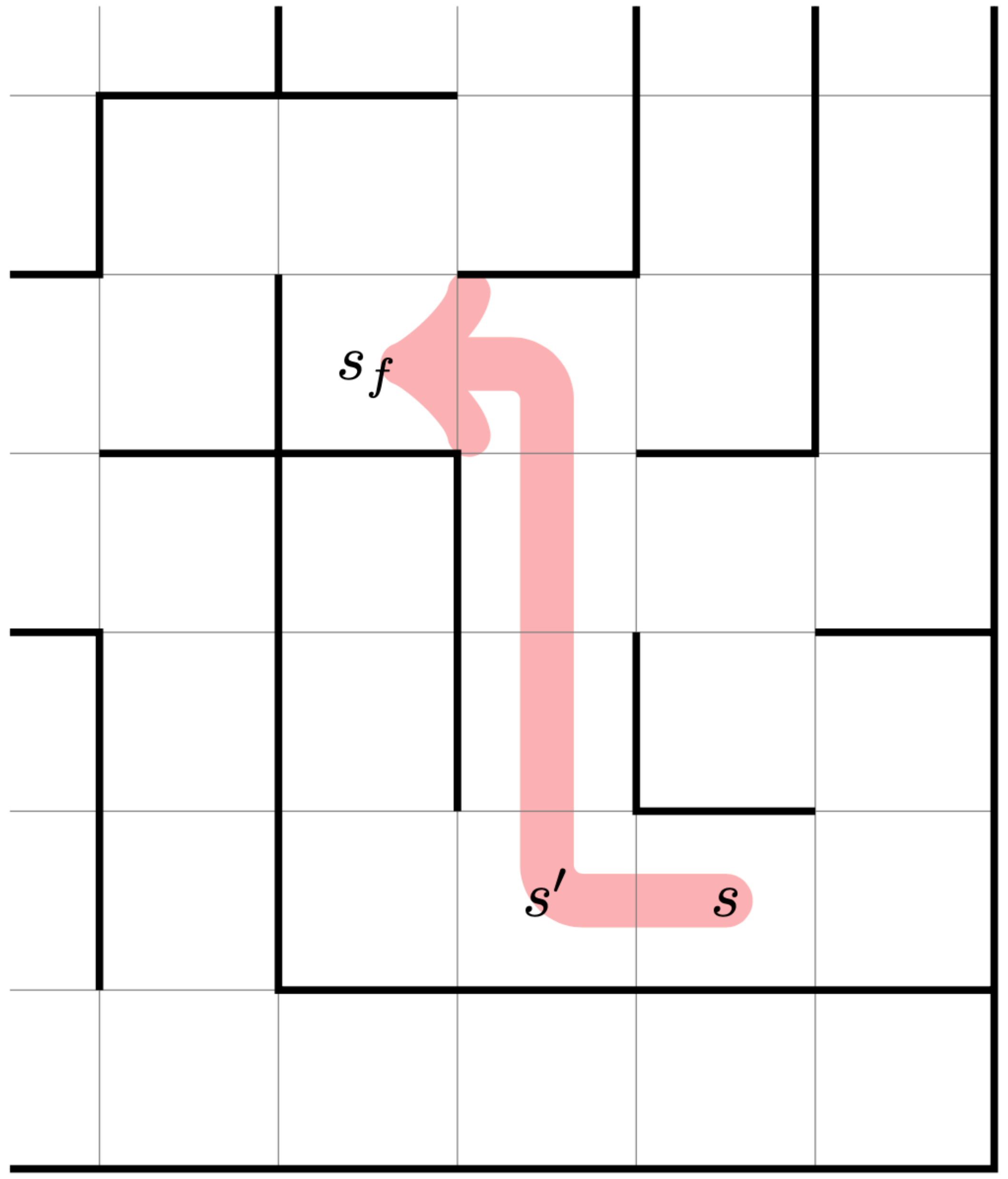
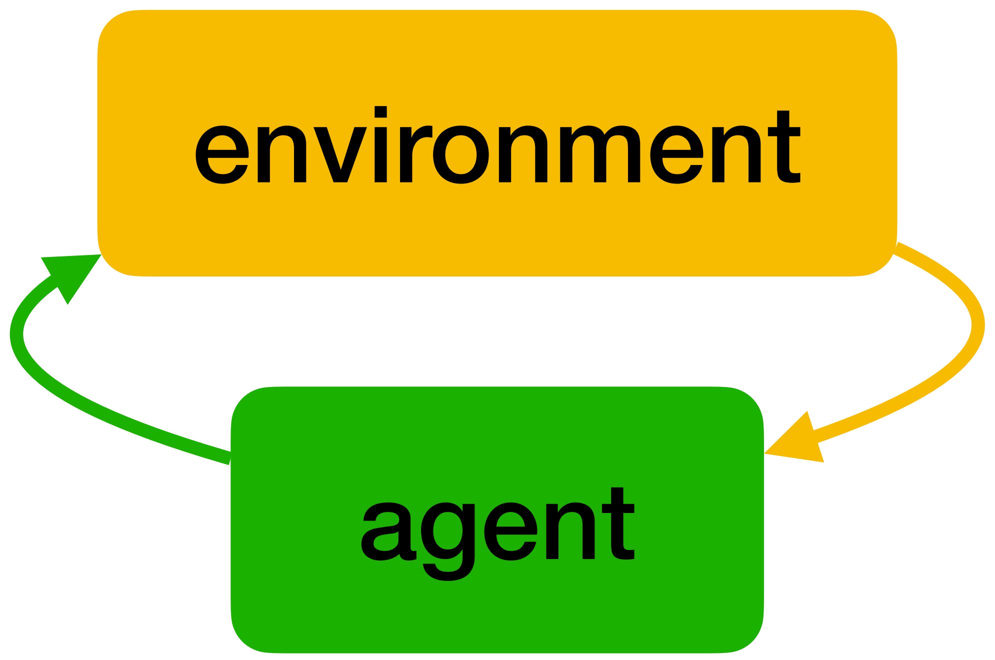
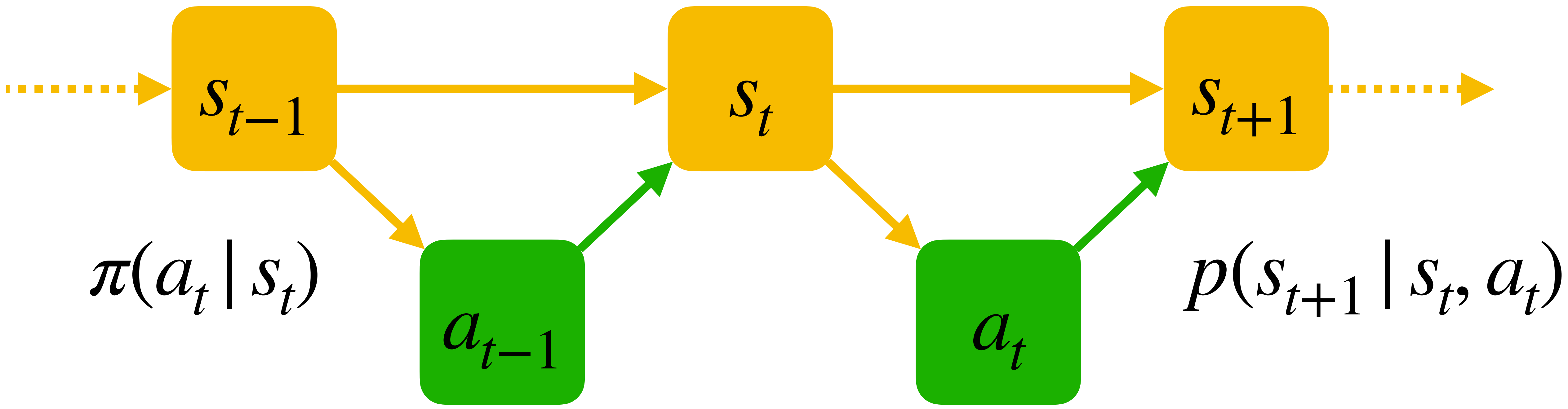
Control preference elicitation

	Explicit	Implicit
"how"	Programming 	Imitation Learning 
"what"	Instruction following 	Reinforcement Learning 

System = agent + environment





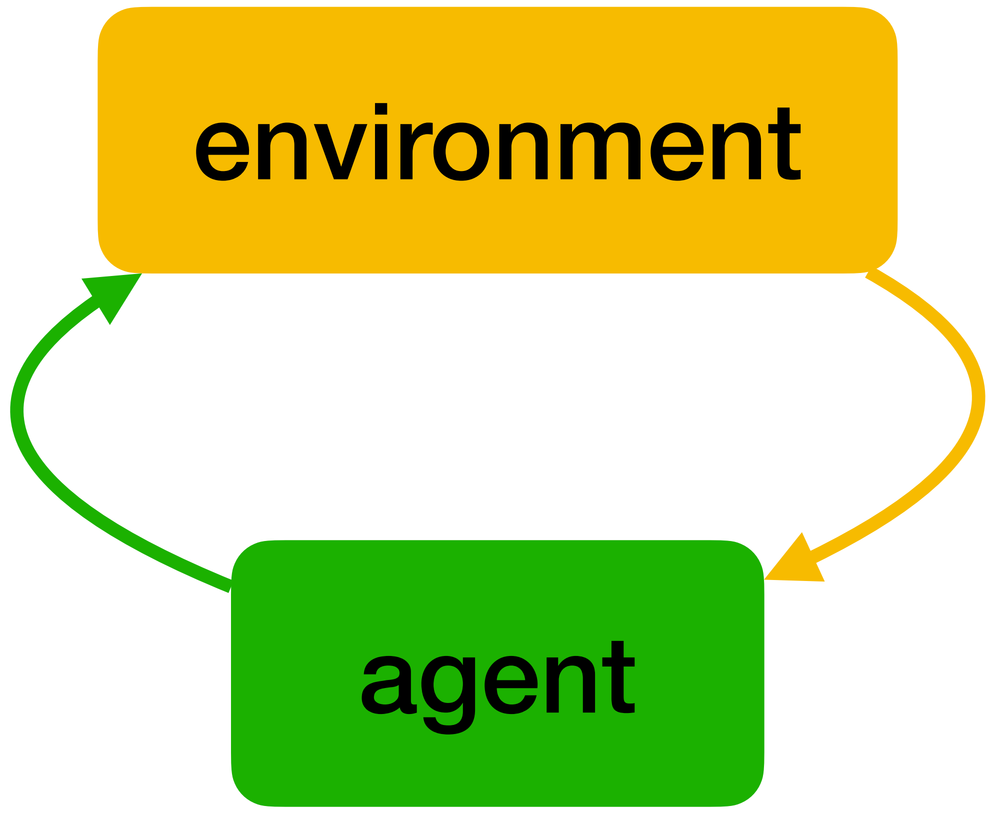


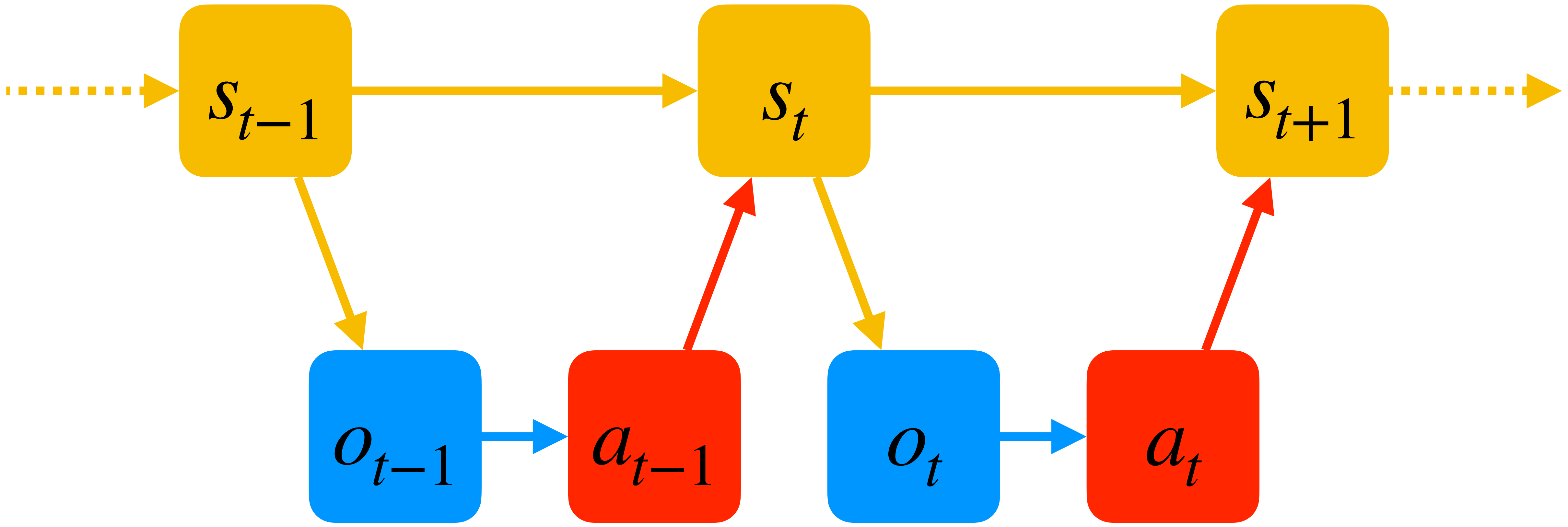
Basic RL concepts

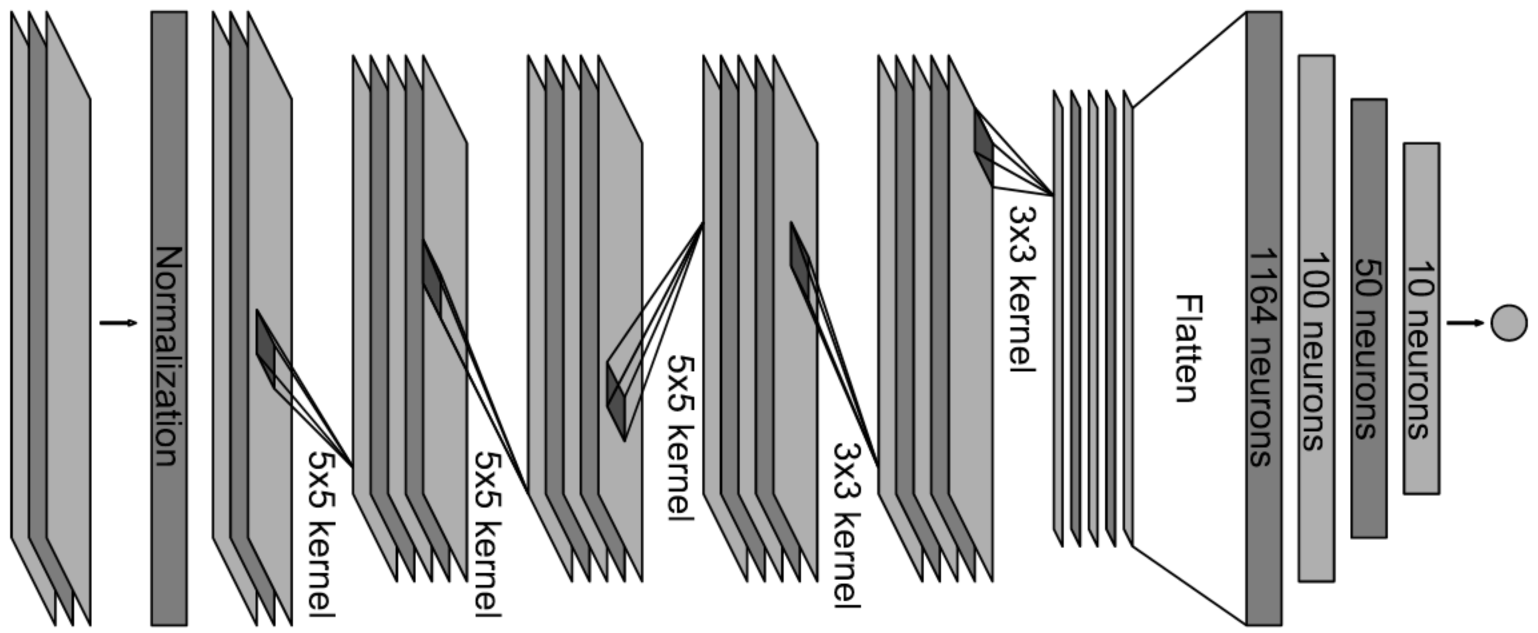
- State: $s \in S$; action: $a \in A$; reward: $r(s, a) \in \mathbb{R}$
- Dynamics: $p(s_{t+1} \mid s_t, a_t)$ for stochastic; $s_{t+1} = f(s_t, a_t)$ for deterministic
- Policy: $\pi(a_t \mid s_t)$ for stochastic; $a_t = \pi(s_t)$ for deterministic
- Trajectory: $p_\pi(\xi = s_0, a_0, s_1, a_1, \ldots) = p(s_0) \prod_t \pi(a_t \mid s_t) p(s_{t+1} \mid s_t, a_t)$
- Return: $R(\xi) = \sum_t \gamma^t r(s_t, a_t) \qquad 0 \leq \gamma < 1$
- Value: $V(s) = \mathbb{E}_{\xi \sim p_\pi}[R \mid s_0 = s]$

  $Q(s, a) = \mathbb{E}_{\xi \sim p_\pi}[R \mid s_0 = s, a_0 = a]$

A policy is a (stochastic) function







$\pi(a_t | o_t)$

observation

action

Horizon classes

- Finite: $R(\xi) = \sum_{t=0}^{T-1} r(s_t, a_t)$
- Infinite: $R(\xi) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} r(s_t, a_t)$
- Discounted: $R(\xi) = \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \qquad 0 \le \gamma < 1$
- Episodic: $R(\xi) = \sum_{t=0}^{T-1} r(s_t, a_t)$ s.t. $s_T = s_f$

Recap

- Marginal state distributions can be computed recursively forward

$$p_\pi(s') = \mathbb{E}_{s \sim p_\pi}[\mathbb{E}_{a|s \sim \pi}[p(s' \mid s, a)]]$$

- Value functions can be computed recursively backward

$$V_\pi(s) = \mathbb{E}_{a|s \sim \pi}[r(s, a) + \gamma \mathbb{E}_{s'|s,a \sim p}[V_\pi(s')]]$$

- Forward and backward recursions are a recurring theme...

Recap

- Trajectory: $\xi = s_0, a_0, s_1, a_1, s_2, \ldots$
- Probability of a trajectory: $p_\pi(\xi) = p(s_0) \prod_t \pi(a_t \,|\, s_t) p(s_{t+1} \,|\, s_t, a_t)$
- Marginal expectation:

$$\mathbb{E}_{\xi \sim p_\pi}[f(s_t)] = \sum_\xi p_\pi(\xi) f(s_t)$$

Recap

- Trajectory: $\xi = s_0, a_0, s_1, a_1, s_2, \dots$
- Probability of a trajectory: $p_\pi(\xi) = p(s_0) \prod_t \pi(a_t | s_t) p(s_{t+1} | s_t, a_t)$
- Marginal expectation:

$$\mathbb{E}_{\xi \sim p_\pi}[f(s_t)] = \sum_\xi p_\pi(\xi) f(s_t)$$

$$= \mathbb{E}_{s_0 \sim p}[\mathbb{E}_{a_0 | s_0 \sim \pi}[\cdots \mathbb{E}_{s_t | s_{t-1}, a_{t-1} \sim p}[\cdots \mathbb{E}_{s_T | s_{T-1}, a_{T-1} \sim p}[f(s_t)] \cdots] \cdots]]$$

$$= \mathbb{E}_{s_0 \sim p}[\mathbb{E}_{a_0 | s_0 \sim \pi}[\cdots \mathbb{E}_{s_t | s_{t-1}, a_{t-1} \sim p}[f(s_t)] \cdots]]$$

$$= \mathbb{E}_{s_t \sim p_\pi}[\mathbb{E}_{\xi | s_t \sim p_\pi}[f(s_t)]] = \mathbb{E}_{s_t \sim p_\pi}[f(s_t)]$$

Recap

- We often take expectations over functions $f(\xi)$ that decompose into a sum
- Finite horizon: $\mathbb{E}_{\xi\sim p_\pi}[f(\xi)] = \mathbb{E}_{\xi\sim p_\pi}\left[\sum_{t=0}^{T-1} f(s_t)\right]$
  - Where $p_\pi(s) = \frac{1}{T}\sum_{t=0}^{T-1} p_\pi(s_t)$, i.e. $p_\pi(s) = \sum_{t=0}^{T-1} p_\pi(t, s_t)$ with $t \sim \mathrm{U}(\{0,\dots,T-1\})$
- Discounted horizon: $\mathbb{E}_{\xi\sim p_\pi}[f(\xi)] = \mathbb{E}_{\xi\sim p_\pi}\left[\sum_{t=0}^{\infty} \gamma^t f(s_t)\right]$
  - Where $p_\pi(s) = (1-\gamma)\sum_{t=0}^{\infty} \gamma^t p_\pi(s_t)$, i.e. $p_\pi(s) = \sum_{t=0}^{\infty} p_\pi(t, s_t)$ with $t \sim \mathrm{Geom}(1-\gamma)$

Recap

- We often take expectations over functions $f(\xi)$ that decompose into a sum
- Finite horizon: $\mathbb{E}_{\xi\sim p_\pi}[f(\xi)] = \mathbb{E}_{\xi\sim p_\pi}\left[\sum_{t=0}^{T-1} f(s_t)\right] = T\mathbb{E}_{s\sim p_\pi}[f(s)]$
  - Where $p_\pi(s) = \frac{1}{T}\sum_{t=0}^{T-1} p_\pi(s_t)$, i.e. $p_\pi(s) = \sum_{t=0}^{T-1} p_\pi(t, s_t)$ with $t \sim \mathrm{U}(\{0,\dots,T-1\})$
- Discounted horizon: $\mathbb{E}_{\xi\sim p_\pi}[f(\xi)] = \mathbb{E}_{\xi\sim p_\pi}\left[\sum_{t=0}^{\infty} \gamma^t f(s_t)\right] = \frac{1}{1-\gamma}\mathbb{E}_{s\sim p_\pi}[f(s)]$
  - Where $p_\pi(s) = (1-\gamma)\sum_{t=0}^{\infty} \gamma^t p_\pi(s_t)$, i.e. $p_\pi(s) = \sum_{t=0}^{\infty} p_\pi(t, s_t)$ with $t \sim \mathrm{Geom}(1-\gamma)$

Taxonomy

- **Imitation Learning**
  - Off-policy
    - BC
    - DART
    - GAIL
  - On-policy
    - DAgger
- **Reinforcement Learning**
  - Temporal Difference
    - DQN
  - Policy Gradient
    - PG
  - DDPG
  - A2C
  - TRPO
- **Model-Based Learning**
  - Planning
    - iLQR
    - MPC
  - MFRL w/ model
    - Dyna

Imitation Learning

- Pros:
  - Teacher action is a globally optimal learning signal
  - Experience distribution is already focused on good behavior
  - Safe
- Cons:
  - Teacher and learner action spaces may mismatch
  - Covariate shift: training distribution $\neq$ test distribution
  - Expensive
  - Teacher may be fallible, inconsistent, etc.

Behavior Cloning (BC)

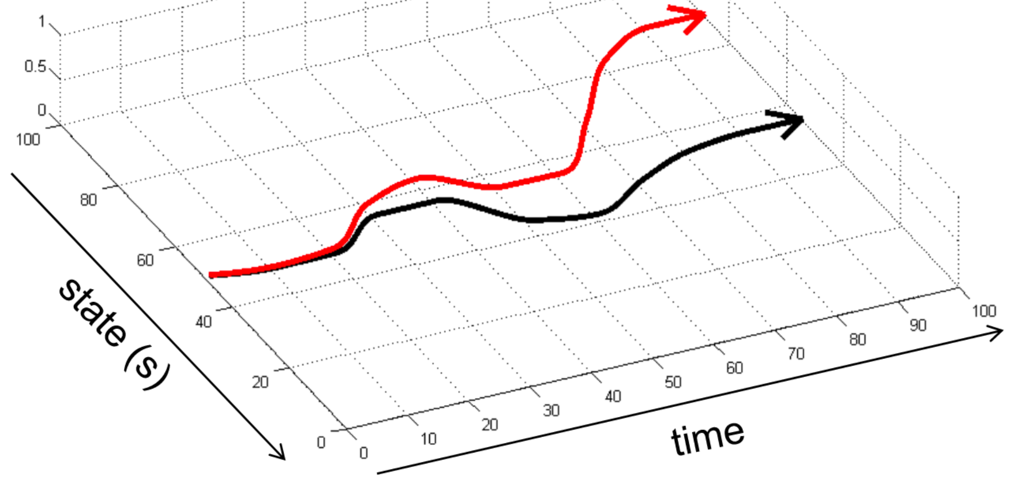
- The simplest IL is just supervised learning:
  - Break trajectories into examples (s_t, a_t)
  - Learn a function $\pi : s \mapsto a$, or a distribution $\pi(a \mid s)$
- One possible loss: negative log-likelihood $\mathscr{L} = -\sum_{(s,a)\in\mathscr{D}} \log \pi(a \mid s)$

Inaccuracy in BC

observations + actions → **training data** → **supervised learning** → $\pi_\theta(a_t | o_t)$

- The state transition distribution is linear in the policy
$$p_\pi(s_{t+1} | s_t) = \sum_{o_t, a_t} p(o_t | s_t)\pi(a_t | o_t)p(s_{t+1} | s_t, a_t)$$

no data here!



- If the policy approximates the teacher $\pi_\theta(a_t | o_t) \approx \pi^*(a_t | o_t)$
  - The dynamics approximates the teacher behavior $p_{\pi_\theta}(s_{t+1} | s_t) \approx p_{\pi^*}(s_{t+1} | s_t)$
- But errors accumulate over time
  - May reach states not seen in the training dataset

Behavior Cloning

- Pros:
  - Simple
  - Data-efficient
- Cons:
  - Very susceptible to train–test mismatch

DAgger: Dataset Aggregation

- Can we collect demonstration data for $p_{\pi_\theta}(o_t)$?

Algorithm 1 DAgger

Collect dataset $\mathcal{D}$ of teacher demonstrations
$(o_0, a_0^*, o_1, a_1^*, \ldots) \sim p_{\pi^*}$
Train π_θ on $\mathcal{D}$
Execute π_θ to get $(o_0, a_0, \ldots) \sim p_{\pi_\theta}$
Ask teacher to label $a_t^* | o_t \sim \pi^*$
Aggregate $(o_0, a_0^*, o_1, a_1^*, \ldots)$ into $\mathcal{D}$
Repeat!

but how? challenging...

- DAgger can reduce the imitation loss from $O(\epsilon T^2)$ to $O(\epsilon T)$

DAgger

- Pros:
  - Addresses covariate shift
  - Good theoretical guarantees
- Cons:
  - Technically challenging: UI for human feedback
  - Burdens human teacher: how to provide good action in weird states, no context

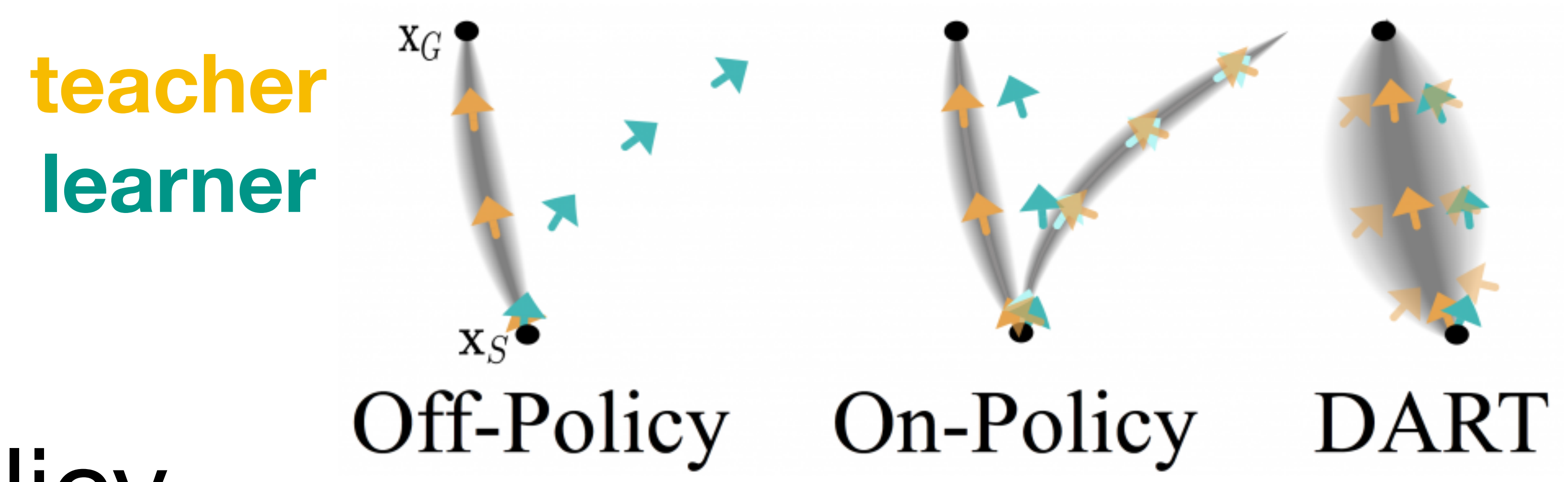
Goal-conditioned Behavior Cloning

- Can we train one policy to reach multiple goals? $\pi_\theta(a_t \mid s_t, g)$
  - Assume goal = state that the agent should reach
- How can we know the goal in demonstrations $\xi = s_0, a_0, s_1, a_1, \ldots$?
  - Require manual labeling?
- Hindsight: take each s_t as the goal of the trajectory leading to it

$$s_0, a_0, \ldots, s_{t-1}, a_{t-1}, s_t = g$$

  - Supervised learning of $\pi(a \mid s, g)$ from data points $(s_t, a_t, s_{t'})$ for $t' > t$

DART: Disturbances Augmenting Robot Training



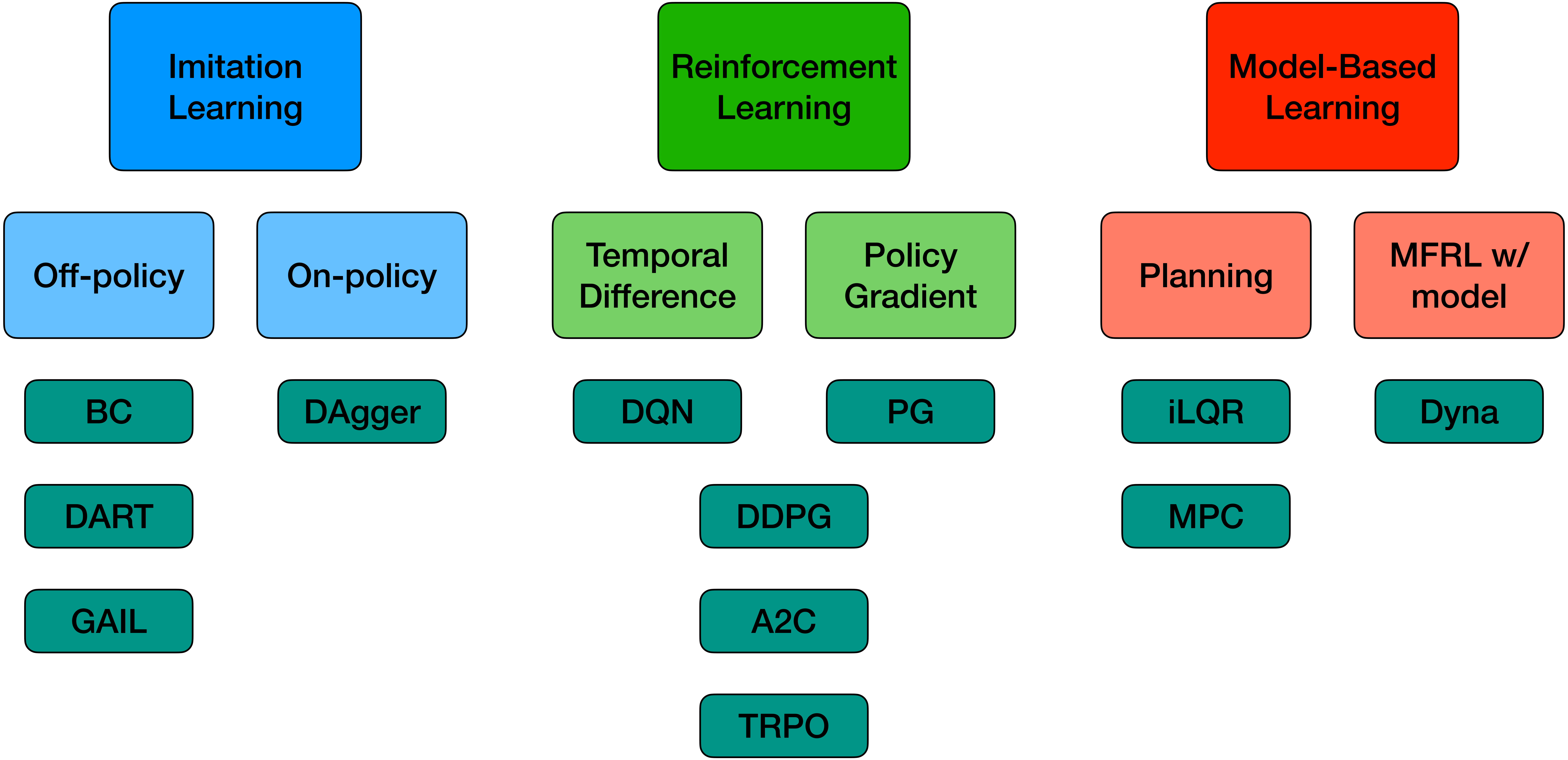
- Off-policy vs. on-policy
  - On-policy = data comes from the learner's current policy
  - Off-policy = data comes from another policy (another agent or past learner)
- In off-policy IL (e.g. BC) learner may go off the teacher's support
- In on-policy IL (e.g. DAgger) learner initially goes off, until corrected
- DART: increase the data support by injecting noise during demonstrations
  - Force teacher into slight-error states, to see how they are fixed

Image: Laskey et al. 2017

DART

- Pros:
  - Addresses covariate shift (to a point)
  - Safe (to a point)
  - Some theoretical guarantees
- Cons:
  - Burdens human teacher: how to provide good action under noise
  - Theoretically optimal noise level may be impossible for human, unsafe

Taxonomy


Imitation Learning
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DQN
PG
iLQR
Dyna
DART
DDPG
MPC
GAIL
A2C
TRPO

Reinforcement Learning

- Pros:
  - Supervisor only provides rewards (“Explicit what”)
  - Very general = lots of applications
- Cons:
  - Data-inefficient
  - Much theory still unknown
  - Much practice unreproducible, many hyperparameters
  - May be unsafe

Recap: policy evaluation

	model-based	model-free
Monte Carlo (MC)	$V_\pi(s_0) = \mathbb{E}_{\xi \sim p_\pi}[R \mid s_0]$	$\xi \sim p_\pi \qquad V(s_0) \to R(\xi)$
Dynamic Programming (DP) / Temporal Difference (TD) (on-policy)	$V_\pi(s) = \mathbb{E}_{a\|s \sim \pi}[r(s,a) + \gamma \mathbb{E}_{s'\|s,a \sim p}[V_\pi(s')]]$	$s, a, r, s' \sim p_\pi \qquad V(s) \to r + \gamma V(s')$
Dynamic Programming (DP) / Temporal Difference (TD) (off-policy)	$Q_\pi(s,a) = r(s,a) + \gamma \mathbb{E}_{\substack{s'\|s,a \sim p \\ a'\|s' \sim \pi}}[Q_\pi(s',a')]]$	$s, a, r, s' \sim p_{\pi'} \quad Q(s,a) \to r + \gamma \mathbb{E}_{a'\|s' \sim \pi}[Q(s',a')]$

Recap: policy ~~evaluation~~ improvement

	model-based	model-free
Monte Carlo (MC)	$V_\pi(s_0) = \mathbb{E}_{\xi \sim p_\pi}[R \mid s_0]$	$\xi \sim p_\pi \qquad V(s_0) \to R(\xi)$
Dynamic Programming (DP) / Temporal Difference (TD) (on-policy)	$V_\pi(s) = \overset{\max\limits_{a}}{\cancel{\mathbb{E}_{a\mid s\sim\pi}}}[r(s,a) + \gamma \mathbb{E}_{s'\mid s,a\sim p}[V_\pi(s')]]$ **Value Iteration**	$s, a, r, s' \sim p_\pi \qquad V(s) \to r + \gamma V(s')$
Dynamic Programming (DP) / Temporal Difference (TD) (off-policy)	$Q_\pi(s,a) = r(s,a) + \gamma \mathbb{E}_{s'\mid s,a\sim p,\ \cancel{a'\mid s'\sim\pi}}[Q_\pi(s',a')]]$ $\max\limits_{a'}$	$s, a, r, s' \sim p_{\pi'} \qquad Q(s,a) \to r + \gamma \overset{\max\limits_{a'}}{\cancel{\mathbb{E}_{a'\mid s'\sim\pi}}}[Q(s',a')]$ **Q-learning**

Deep TD reinforcement learning

- Deep Q Learning (historically called DQN):

$$\mathscr{L}_\theta(s, a, r, s') = (r + \gamma \max_{a'} Q_{\bar{\theta}}(s', a') - Q_\theta(s, a))^2$$

- This algorithm should work off-policy, so we can keep past experience
  - Replay buffer = data set of recent past experience of learner policy at that time
  - Variants differ on
    - How to add experience to the buffer
    - How to sample from the buffer

Temporal Difference

- Pros:
  - Data-efficient (Dynamic Programming)
  - Off-policy
  - Biologically inspired, some evidence in neuroscience (dopamine $\approx$ Bellman error)
- Cons:
  - Learns value function, not a policy
  - Much is unknown about optimal exploration

Policy Gradient (PG)

- Unlike minimizing $\mathscr{L}_\theta(\mathscr{D})$ in general ML, in RL we maximize $\mathscr{J}_\theta = \mathbb{E}_{\xi \sim p_{\pi_\theta}}[R]$
- This is harder since the “data” distribution depends on θ
- But there's a trick: $\nabla_\theta \log p_\theta(\xi) = \dfrac{1}{p_\theta(\xi)} \nabla_\theta p_\theta(\xi)$
- Log-derivative / score-function / REINFORCE trick: estimate gradient using samples of $p_\theta(\xi)$

$$
\begin{aligned}
\nabla_\theta \mathscr{J}_\theta &= \nabla \theta \int d\xi p_\theta(\xi) R(\xi) \\
&= \int d\xi p_\theta(\xi) \nabla_\theta \log p_\theta(\xi) R(\xi) \\
&= \mathbb{E}_{\xi \sim p_\theta}[\nabla_\theta \log p_\theta(\xi) R]
\end{aligned}
$$

REINFORCE (1992 !)

- Roll out π_θ to sample $\xi \sim p_\theta$
- Compute $R(\xi)$ and

$$\nabla_\theta \log p_\theta(\xi) = \nabla_\theta(\log p(s_0) + \sum_t (\log \pi_\theta(a_t \mid s_t) + \log p(s_{t+1} \mid s_t, a_t)))$$

- Take a gradient step with $\nabla_\theta \log p_\theta(\xi) R$
- Repeat

- This is model-free! but on-policy, + high variance of the gradient estimator

Baselines

- Constant shifts in return shouldn't matter for optimal policy

$$0 = \nabla_\theta \mathbb{E}_{\xi \sim p_\theta}[b] = \mathbb{E}_{\xi \sim p_\theta}[\nabla_\theta \log p_\theta(\xi) b]$$

- Can we use that to reduce variance without adding bias?

- Using the average return works pretty well in practice

$$\nabla_\theta \mathcal{J}_\theta \approx \frac{1}{N} \sum_i \nabla_\theta \log p_\theta(\xi_i)(R_i - b)$$

- With $b = \frac{1}{N} \sum_i R_i$

Don't let the past distract you

$$\nabla_\theta \mathcal{J}_\theta = \mathbb{E}_{\xi \sim p_\theta}\left[\left(\sum_t \nabla_\theta \log \pi_\theta(a_t | s_t)\right) R\right] = \sum_t \mathbb{E}_{s_t \sim p_\theta}[\nabla_\theta \mathbb{E}_{a_t | s_t \sim \pi_\theta}[R]]$$

- In our case, $R_{\geq t} = \sum_{t' \geq t} \gamma^{t'} r(s_{t'}, a_{t'})$ is a sufficient statistic of R for $\mathcal{J}_\theta$
- Therefore, a lower-variance gradient estimator:

$$\sum_t \gamma^t \mathbb{E}_{s_t \sim p_\theta}[\mathbb{E}_{a_t | s_t \sim \pi_\theta}[\nabla_\theta \log \pi_\theta(a_t | s_t) R_{\geq t}]]$$

Policy-Gradient Theorem

$$\nabla_\theta \mathcal{J}_\theta = \sum_t \gamma^t \mathbb{E}_{s_t \sim p_\theta}[\mathbb{E}_{a_t|s_t \sim \pi_\theta}[\nabla_\theta \log \pi_\theta(a_t|s_t) R_{\geq t}]]$$

$$\stackrel{!}{=} \sum_t \gamma^t \mathbb{E}_{s_t \sim p_\theta}[\mathbb{E}_{a_t|s_t \sim \pi_\theta}[\nabla_\theta \log \pi_\theta(a_t|s_t) Q_{\pi_\theta}(s_t, a_t)]]$$

$$\nabla_\theta V_{\pi_\theta}(s) = \nabla_\theta \mathbb{E}_{a|s \sim \pi_\theta}[Q_{\pi_\theta}(s, a)]$$

$$= \sum_a (\nabla_\theta \pi_\theta(a|s) Q_{\pi_\theta}(s, a) + \pi_\theta(a|s) \nabla_\theta Q_{\pi_\theta}(s, a))$$

$$= \mathbb{E}_{a|s \sim \pi_\theta}[\nabla_\theta \log \pi_\theta(a|s) Q_{\pi_\theta}(s, a) + \nabla_\theta(r(s, a) + \gamma \mathbb{E}_{s'|s,a \sim p}[V_{\pi_\theta}(s')])]$$

$$= \mathbb{E}_{a|s \sim \pi_\theta}[\nabla_\theta \log \pi_\theta(a|s) Q_{\pi_\theta}(s, a) + \gamma \mathbb{E}_{s'|s,a}[\nabla_\theta V_{\pi_\theta}(s')]]$$

- Here back-propagating gradients is like a Bellman recursion

DDPG

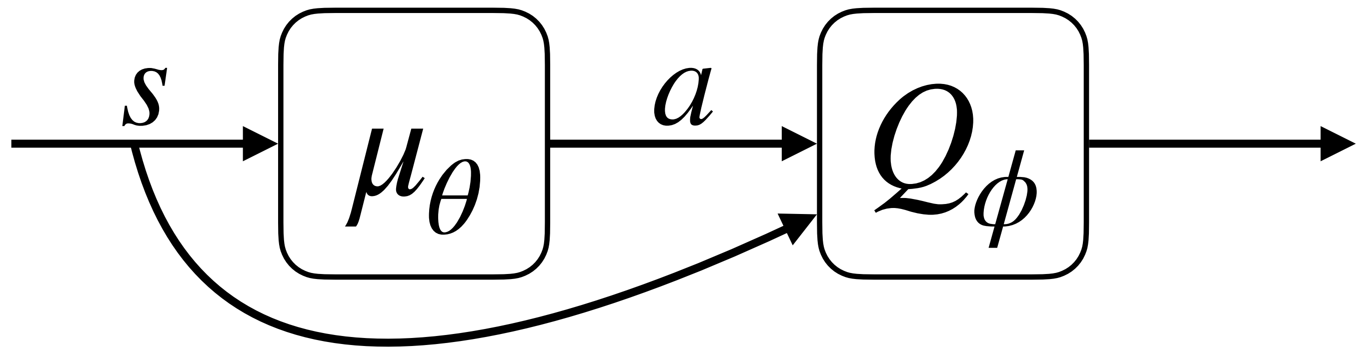
- Previous methods: represent a Q maximizer or train one ad-hoc
- More general method: let a deterministic $\mu_\theta(s)$ learn to maximize $Q_\phi(s, a)$
  - This makes it an Actor–Critic method; critic trained with any TD method
- Policy Gradient Theorem:

$$\nabla_\theta \mathcal{J}_\theta = \mathbb{E}_{s,a \sim p_\theta}[\nabla_\theta \log \pi_\theta(a \mid s) Q_{\pi_\theta}(s, a)]$$

- Deterministic Policy Gradient Theorem:

$$\nabla_\theta \mathcal{J}_\theta = \mathbb{E}_{s \sim p_\theta}\left[\nabla_\theta \mu_\theta(s) \nabla_a Q_{\mu_\theta}(s, a)\Big|_{a=\mu_\theta(s)}\right]$$

$\leftarrow \nabla_\theta \mu_\theta(s) \quad \nabla_a Q_\phi(s, a) \leftarrow$



Baselines

$$\nabla_\theta \mathcal{J}_\theta = \mathbb{E}[\nabla_\theta \log \pi_\theta(a|s)(Q_{\pi_\theta}(s,a) - b) | s, a]$$

- b can be any variable independent of a given s
  - Can depend on the past, but not the future
- Previously, we used the average as baseline $b = \frac{1}{N}\sum_i R_i$
- This suggests using the state value as baseline $b = V_{\pi_\theta}(s)$

Advantage estimation

$$\nabla_\theta \mathcal{J}_\theta = \mathbb{E}[\nabla_\theta \log \pi_\theta(a \mid s) A_{\pi_\theta}(s, a) \mid s, a]$$

- How to estimate $A_{\pi_\theta}(s, a)$?

$$A_\pi(s, a) = Q_\pi(s, a) - V_\pi(s) = r(s, a) + \gamma \mathbb{E}_{s' \mid s, a \sim p}[V_\pi(s')] - V_\pi(s)$$

- With value estimation $V_\phi(s)$, estimate the advantage:

$$\hat{A}(s, a) \approx r + \gamma V_\phi(s') - V_\phi(s)$$

An Actor–Critic algorithm

Algorithm 1 Actor–Critic

get on-policy sample (s, a, r, s')
take gradient step on $\mathcal{L}_\phi = (r + \gamma V_{\bar{\phi}}(s') - V_\phi(s))^2$
compute $\hat{A}(s, a) = r + \gamma V_\phi(s') - V_\phi(s)$
take gradient step $\nabla_\theta \log \pi_\theta(a|s) \hat{A}(s, a)$
repeat

Policy Gradient algorithms

- REINFORCE: $\nabla_\theta \mathcal{J}_\theta \approx \nabla_\theta \log \pi_\theta(a_t | s_t) R$
- Baselines (= control variates): $\nabla_\theta \mathcal{J}_\theta \approx \nabla_\theta \log \pi_\theta(a_t | s_t)(R - b)$
- Future return (Rao–Blackwell): $\nabla_\theta \mathcal{J}_\theta \approx \nabla_\theta \log \pi_\theta(a_t | s_t) R_{\geq t}$
- Policy Gradient Theorem: $\nabla_\theta \mathcal{J}_\theta \approx \left(\nabla_\theta \log \pi_\theta(a_t | s_t)\right) Q_\theta(s_t, a_t)$
- Actor–Critic: $\nabla_\theta \mathcal{J}_\theta \approx \nabla_\theta \log \pi_\theta(a_t | s_t) \hat{A}(s_t, a_t)$
  - Algorithms differ by how they estimate advantages

Comparing advantage estimators

	bias	variance
$\nabla_\theta \mathcal{J}_\theta \approx \nabla_\theta \log \pi_\theta(a\,|\,s)\left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b\right)$	none	high one grad per traj
$\nabla_\theta \mathcal{J}_\theta \approx \nabla_\theta \log \pi_\theta(a\,|\,s)(r + \gamma V_\phi(s') - V_\phi(s))$	some approx value	lower
$\nabla_\theta \mathcal{J}_\theta \approx \nabla_\theta \log \pi_\theta(a\,|\,s)\left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - V_\phi(s)\right)$	none	mid state-dependent baseline

n-step TD

- 1-step TD: $\hat{A}_t^1 = r_t + \gamma V(s_{t+1}) - V(s_t)$
- 2-step TD: $\hat{A}_t^2 = r_t + \gamma r_{t+1} + \gamma^2 V(s_{t+2}) - V(s_t)$

...

- n-step TD: $\hat{A}_t^n = r_t + \cdots + \gamma^{n-1} r_{t+n-1} + \gamma^n V(s_{t+n}) - V(s_t)$
- In the limit (MC): $\hat{A}_t^\infty = r_t + \gamma r_{t+1} + \cdots - V(s_t)$

Generalized Advantage Estimation (GAE(λ))

$$\nabla_\theta \mathcal{J}_\theta \approx \nabla_\theta \log \pi_\theta(a_t \mid s_t) \sum_{t'} (\lambda\gamma)^{t'} \hat{A}^1_{t+t'}$$

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$$\hat{A}^1_t = r_t + \gamma V(s_{t+1}) - V(s_t)$$

- GAE(0) = 1-step; GAE(1) = MC

Off-policy MC advantage estimation

- MC advantage estimation: $\sum_t \gamma^t r(s_t, a_t) - V_{\pi_\theta}(s)$

$$= \sum_t \gamma^t (r(s_t, a_t) + \gamma V_{\pi_\theta}(s_{t+1}) - V_{\pi_\theta}(s_t)) = \sum_t \gamma^t \hat{A}^1_{\pi_\theta}(s_t, a_t)$$

- What if estimate the advantage of our current π_θ under a proposed new policy $\pi_{\theta'}$?

we want to maximize this!

$$\mathbb{E}_{\xi \sim p_{\theta'}}\left[\sum_t \gamma^t \hat{A}^1_{\pi_\theta}(s_t, a_t)\right] = \mathbb{E}_{\xi \sim p_{\theta'}}\left[\sum_t \gamma^t r(s_t, a_t) - V_{\pi_\theta}(s_0)\right] = \mathcal{J}_{\theta'} - \mathcal{J}_\theta$$

$$= \sum_t \gamma^t \mathbb{E}_{s_t, a_t \sim p_{\theta'}}[\hat{A}^1_{\pi_\theta}(s_t, a_t)]$$

$$= \sum_t \gamma^t \mathbb{E}_{s_t \sim p_{\theta'}}\left[\mathbb{E}_{a_t | s_t \sim \pi_\theta}\left[\frac{\pi_{\theta'}(a_t | s_t)}{\pi_\theta(a_t | s_t)} \hat{A}^1_{\pi_\theta}(s_t, a_t)\right]\right]$$

  - We can't estimate this empirically, because of $s_t \sim p_{\theta'}$; what's the difference if we just use $s_t \sim p_\theta$?

Trust-Region Policy Optimization (TRPO)

$$\max_{\theta'} \sum_t \gamma^t \mathbb{E}_{s_t \sim p_\theta} \left[\mathbb{E}_{a_t|s_t \sim \pi_\theta} \left[\frac{\pi_{\theta'}(a_t \,|\, s_t)}{\pi_\theta(a_t \,|\, s_t)} \hat{A}^1_{\pi_\theta}(s_t, a_t) \right] \right]$$

s.t. $\mathbb{D}[\pi_{\theta'} \| \pi_\theta] \leq \epsilon$

- For small enough ϵ, the objective is close to $\mathcal{J}_{\theta'} - \mathcal{J}_\theta$
  - Guarantees improvement of our objective; in practice, good ϵ is too large
- TRPO loss:

importance weight

advantage estimate

Lagrange multiplier for KL constraint

$$\mathcal{L}_\theta(s, a, r, s') = -\frac{\pi_\theta(a \,|\, s)}{\pi_{\bar{\theta}}(a \,|\, s)}(r + \gamma V_\phi(s') - V_\phi(s)) + \lambda \mathbb{D}[\pi_\theta(\,\cdot\,|\,s) \| \pi_{\bar{\theta}}(\,\cdot\,|\,s)]$$

- The actual algorithm is more complicated; simpler variant: PPO

Taxonomy

- Imitation Learning
  - Off-policy
    - BC
    - DART
    - GAIL
  - On-policy
    - DAgger
- Reinforcement Learning
  - Temporal Difference
    - DQN
  - Policy Gradient
    - PG
  - DDPG
  - A2C
  - TRPO
- Model-Based Learning
  - Planning
    - iLQR
    - MPC
  - MFRL w/ model
    - Dyna

Iterative LQR (iLQR)

Algorithm 1 iLQR

linearize dynamics around current trajectory $(\hat{x}, \hat{u})$

compute $A, B \leftarrow \nabla_x \hat{f}_t, \nabla_u \hat{f}_t$

quadratic cost approximation around $(\hat{x}, \hat{u})$

compute $Q, R, N, q, r \leftarrow \nabla_x^2 \hat{c}_t, \nabla_u^2 \hat{c}_t, \nabla_{xu} \hat{c}_t, \nabla_x \hat{c}_t, \nabla_u \hat{c}_t$

$\hat{L}_t, \hat{\ell}_t \leftarrow$ LQR on $\delta x_t = x_t - \hat{x}_t$, $\delta u_t = u_t - \hat{u}_t$ ← solve with LQR

$\delta x^*, \delta u^* \leftarrow$ execute policy $\delta u_t = \hat{L}_t \delta x_t + \hat{\ell}_t$ in the simulator / environment

$\hat{x} \leftarrow \hat{x} + \delta x^*$, $\hat{u} \leftarrow \hat{u} + \delta u^*$

roll out to get new trajectory $(\hat{x}, \hat{u})$

repeat to convergence

Model-free RL with a model

- General scheme for using a model for model-free RL:

$$
\begin{aligned}
&\text{collect data} \\
&\text{train model } \hat{p}, \hat{r} \\
&\textbf{repeat} \\
&\quad \text{sample } s \text{ from the replay buffer} \\
&\quad \text{sample } a|s \text{ from the learner's policy (or anything else)} \\
&\quad \text{simulate } r = \hat{r}(s,a) \text{ and } s'|s,a \sim \hat{p} \\
&\quad \text{perform model-free RL with } (s,a,r,s')
\end{aligned}
$$

interaction with environment (random policy) (collect data)

supervised learning (train model)

seeded by initial interaction
may interact more as learner improves (replay buffer)

use model as simulator (simulate)

- Benefit: get diverse off-policy s, and fresh on-policy a

Issues with approximate models

- Model inaccuracy accumulates
  - If $|p_\phi(s'|s,a) - p(s'|s,a)|_1 \leq \epsilon$ then $|p_\phi(s_t) - p(s_t)|_1 \leq \epsilon t$
  - We have to plan far enough ahead to realize the consequences of actions
  - But we don't have to execute those plans far ahead!
- Model Predictive Control (MPC):

$\mathcal{D} \leftarrow$ collect data
repeat
    $\hat{\mathcal{M}} \leftarrow$ train model $\hat{p}, \hat{r}$ from $\mathcal{D}$
    repeat
        $\pi \leftarrow$ plan in $\hat{\mathcal{M}}$ from current state s to horizon H
        take *one action* a according to π
        add empirical (s, a, r, s') to $\mathcal{D}$

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