CS 277: Control and Reinforcement Learning

Winter 2024

Lecture 12: Partial Observability

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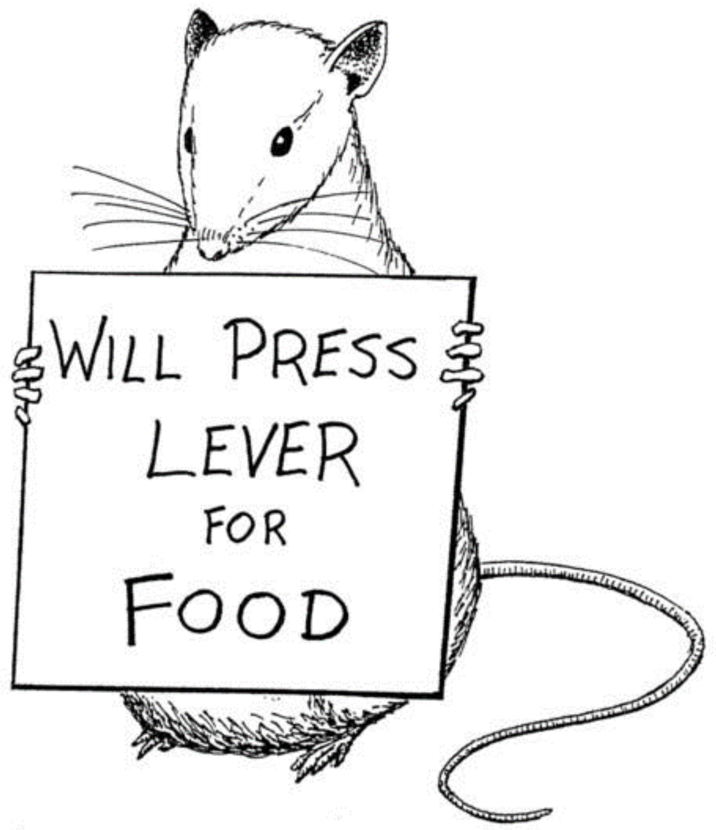


Logistics

assignments

- Exercise 3 due next Monday
- Quiz 6 to be published soon, due next Wednesday

Today's lecture

Partially Observable MDPs

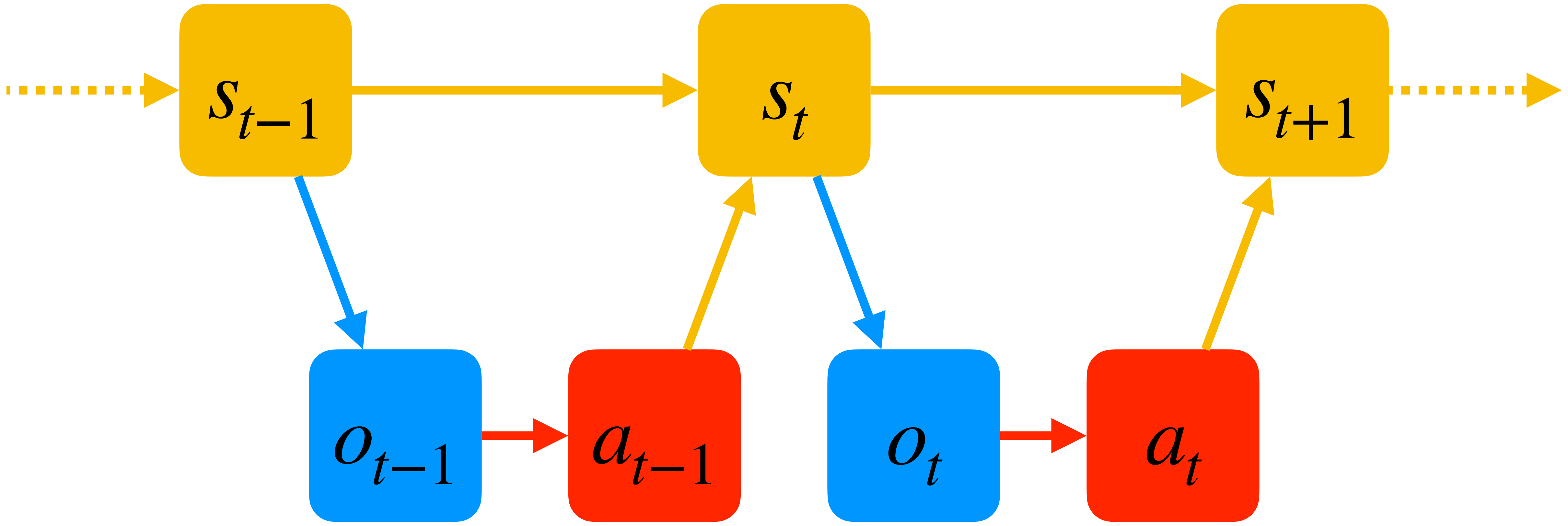
Belief-state MDPs

RNNs

What does the policy depend on?

- Minimally: nothing
  - Just an open-loop sequence of actions $a_0, a_1, \dots$
    - Except, even this depends on a clock $a_t = \pi(t)$
- Typically: the current state $\pi(a_t \mid s_t)$
- What if the state is not fully observable to the agent's sensors?
  - Completely unobservable → forced open loop
  - Partially observable → $\pi(a_t \mid o_t)$?

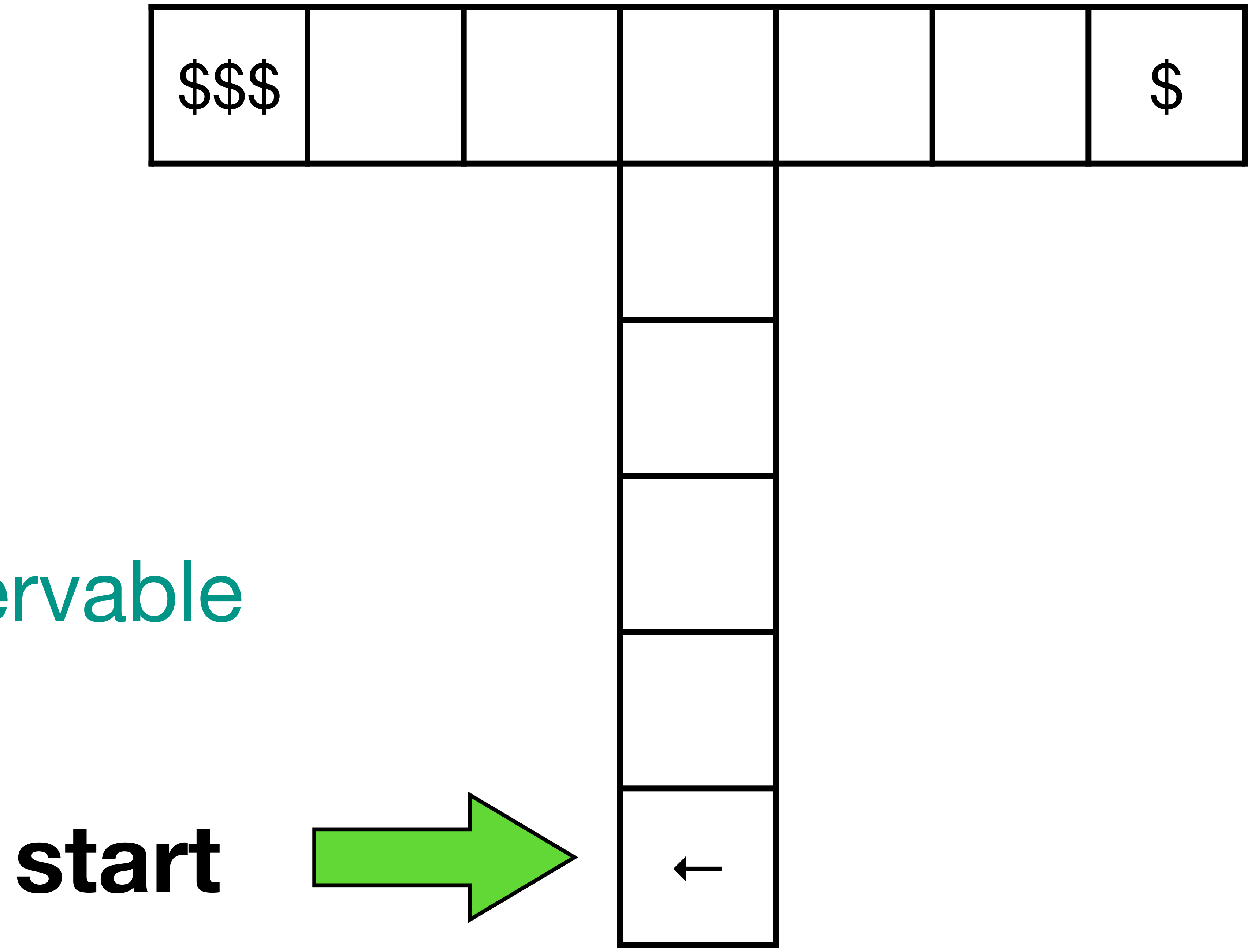
Partially Observable Markov Decision Process (POMDP)



- States $\mathscr{S}$
- Actions $\mathscr{A}$
- Observations $\mathscr{O}$
- Transitions $p(s_{t+1} | s_t, a_t)$
- Emissions (observation model) $p(o_t | s_t)$
- Rewards $r(s_t, a_t)$

T-maze domain

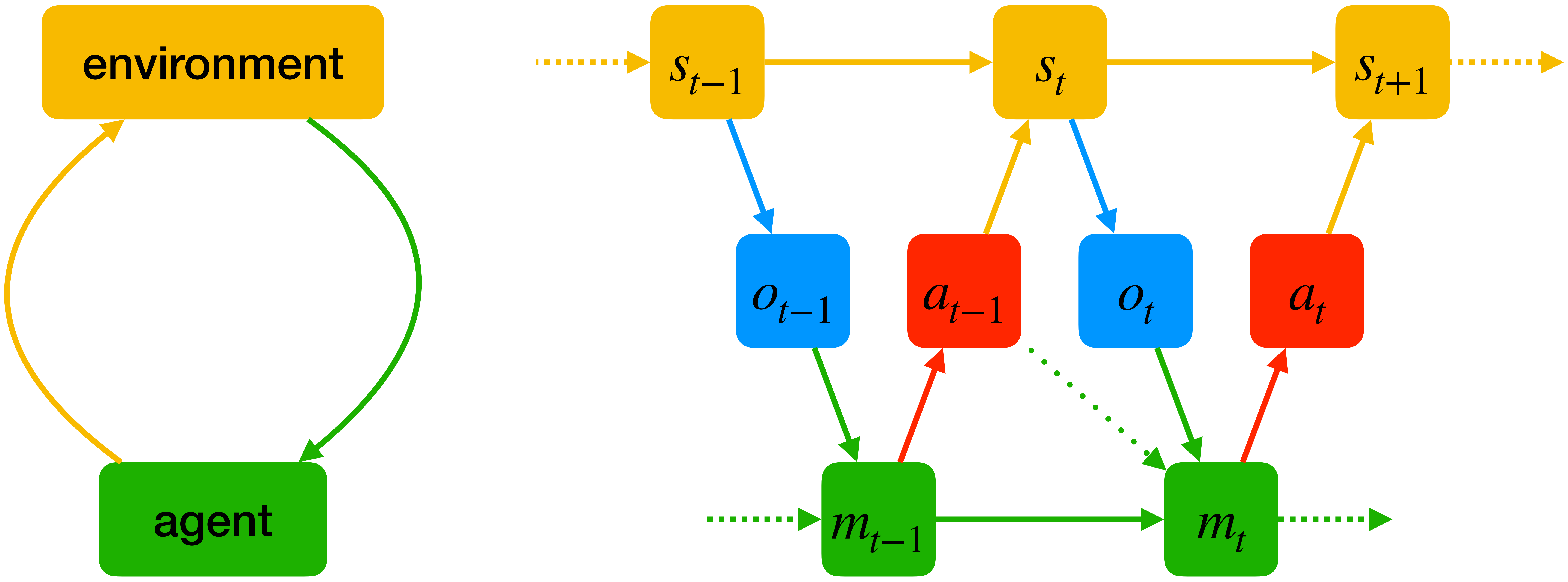
- Observation: current cell
- Observe cue at start
  - Decision at T-junction — cue no longer observable
- Memory is needed



What does the policy depend on? (revisited)

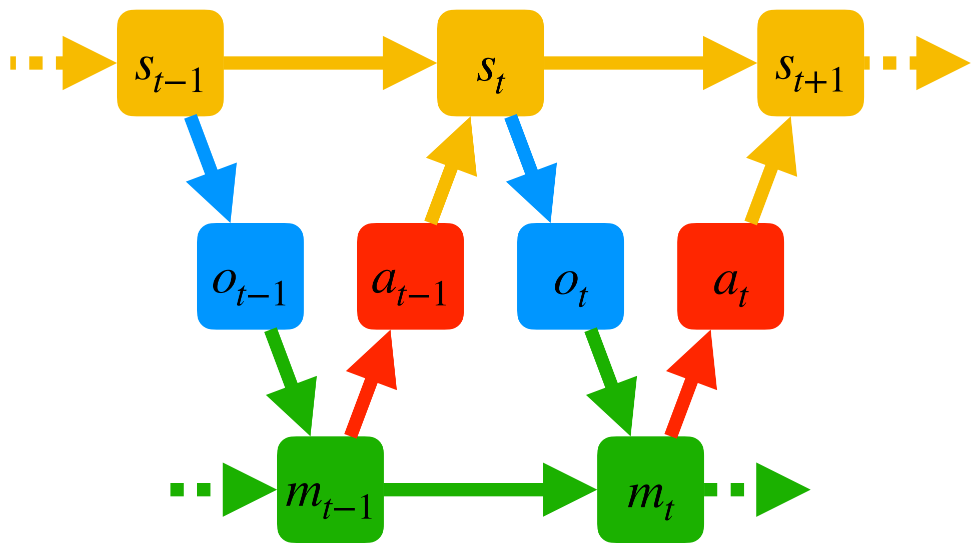
- Maximally: the entire observable history $\pi(a_t | h_t = (o_0, o_1, \ldots, o_t))$
  - Should we remember past actions?
    - In a stochastic policy $\pi(a_t | h_t)$, yes: $h_t = (o_0, a_0, o_1, a_1, \ldots, o_t)$
    - In a deterministic policy $\pi : h_t \mapsto a_t$, we could regenerate a_{t-1} from h_{t-1} (more compute)
- Problem: we can't have unbounded memory that grows with t
- Solution 1: keep a window of k last observations $\pi(a_t | o_{t-k+1}, \ldots, o_t)$ (frame stacking)
- Solution 2: keep a statistic $m_t = \pi(h_t)$ or $\pi(m_t | h_t)$ of the observable history, use $\pi(a_t | m_t)$
  - Memory must allow sequential updates: $m_t = f(m_{t-1}, o_t)$ or $m_t = f(m_{t-1}, a_{t-1}, o_t)$

Agent–environment interaction



- Agent policy: $\pi(m_t, a_t | m_{t-1}, o_t) = \pi(m_t | m_{t-1}, o_t)\pi(a_t | m_t)$
- For simplicity, no edge $a_{t-1} \to m_t$
  - Can make a_{t-1} explicitly observable in o_t, or explicitly remembered in m_{t-1}

So what is memory?



- There's no Markov property in the observable process alone
  - All past observations may be informative of future actions
- Filter the observable past to provide more information about the hidden state
- No less important: plan for the future
- Previously, we needed to trade off short-term with long-term rewards
  - Now we also need to trade off with information-gathering = active perception
- In multi-agent: state of the world is incomplete without other agent's memory
  - Theory of mind

Tiger domain

- 2 states: which door leads to a tiger (-100 reward) and which to $$$ (+10)
- You can stop and listen: $p(o_t = s_t \mid s_t) = 0.8$

$p(s_0 = s_{\text{left}}) = 0.5$ $\quad\mathbb{E}[r(s_0, a_{\text{left}})] = -45$ $\quad o_1 = o_{\text{right}}$

$p(s_1 = s_{\text{left}}) = 0.2$ $\quad\mathbb{E}[r(s_1, a_{\text{left}})] = -12$ $\quad o_2 = o_{\text{left}}$

$p(s_2 = s_{\text{left}}) = 0.5$ $\quad\mathbb{E}[r(s_2, a_{\text{left}})] = -45$ $\quad o_3 = o_{\text{right}}$

$p(s_3 = s_{\text{left}}) = 0.2$ $\quad\mathbb{E}[r(s_3, a_{\text{left}})] = -12$ $\quad o_4 = o_{\text{right}}$

$p(s_4 = s_{\text{left}}) = \frac{0.04}{0.04 + 0.64} \approx 0.06$ $\quad\mathbb{E}[r(s_4, a_{\text{left}})] = -3.5$ $\quad o_5 = o_{\text{right}}$

$p(s_5 = s_{\text{left}}) \approx 0.015$ $\quad\mathbb{E}[r(s_4, a_{\text{left}})] = -8.3$

Today's lecture

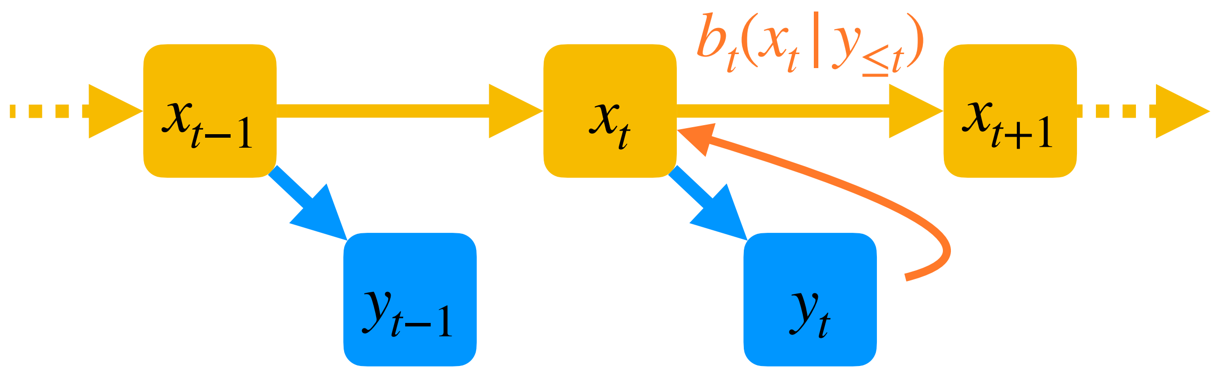
Partially Observable MDPs

Belief-state MDPs

RNNs

Belief

- Belief = distribution over the state $b(s)$
  - If the agent reaches belief b after history h, that does not imply $s \sim b$
- Bayesian belief $b_h(s) = p(s \mid h)$: a sufficient statistic of h for s
  - For a Bayesian belief: $s \sim b_h$ after history h
- In the linear–Gaussian case: the Kalman filter



  - Bayesian belief is Gaussian $p(x_t \mid h_t = y_{\leq t}) = \mathcal{N}(x_t; \hat{x}_t, \Sigma_t)$
  - Covariance can be precomputed $\mathbb{V}(x_t \mid h_t) = \Sigma_t$ (independent of h_t)
  - Mean can be updated linearly: $\hat{x}'_t = A\hat{x}_{t-1} + Bu_{t-1}$ $\quad e_t = y_t - C\hat{x}'_t$ $\quad \hat{x}_t = \hat{x}'_t + K_t e_t$

Computing the Bayesian belief

- Predict s_{t+1} from $h_t = (o_0, a_0, o_1, a_1, \dots, o_t)$ and a_t:

$$b'_t(s_{t+1} | h_t, a_t) = \sum_{s_t} p(s_t | h_t) p(s_{t+1} | s_t, a_t) = \sum_{s_t} b_t(s_t) p(s_{t+1} | s_t, a_t)$$

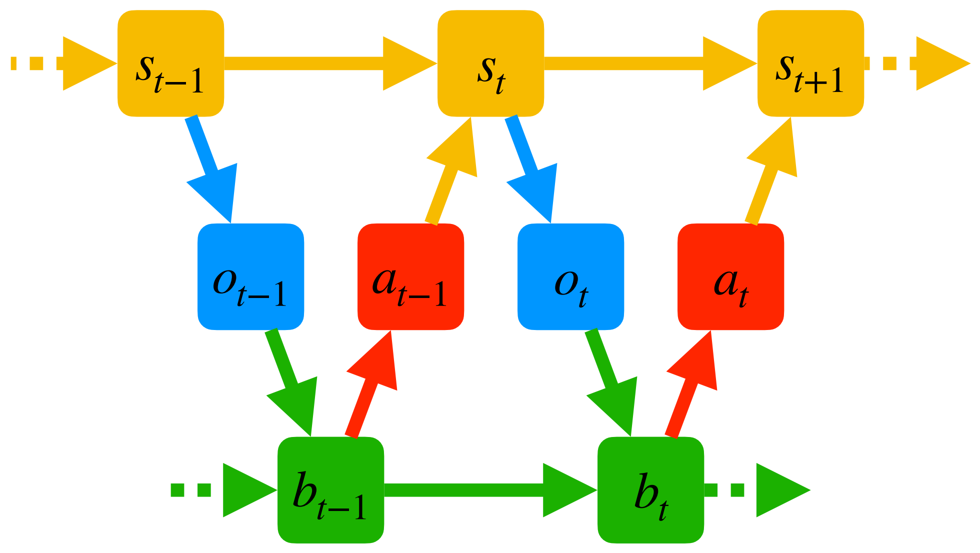
total probability over s_t · previous belief b_t · dynamics needs to be known

- Update belief of s_t after seeing $h_t = (h_{t-1}, a_{t-1}, o_t)$:

$$b_t(s_t | h_t) = \frac{p(s_t | h_{t-1}, a_{t-1}) p(o_t | s_t)}{p(o_t | h_{t-1}, a_{t-1})} = \frac{b'_{t-1}(s_t) p(o_t | s_t)}{\sum_{\bar{s}_t} b'_{t-1}(\bar{s}_t) p(o_t | \bar{s}_t)}$$

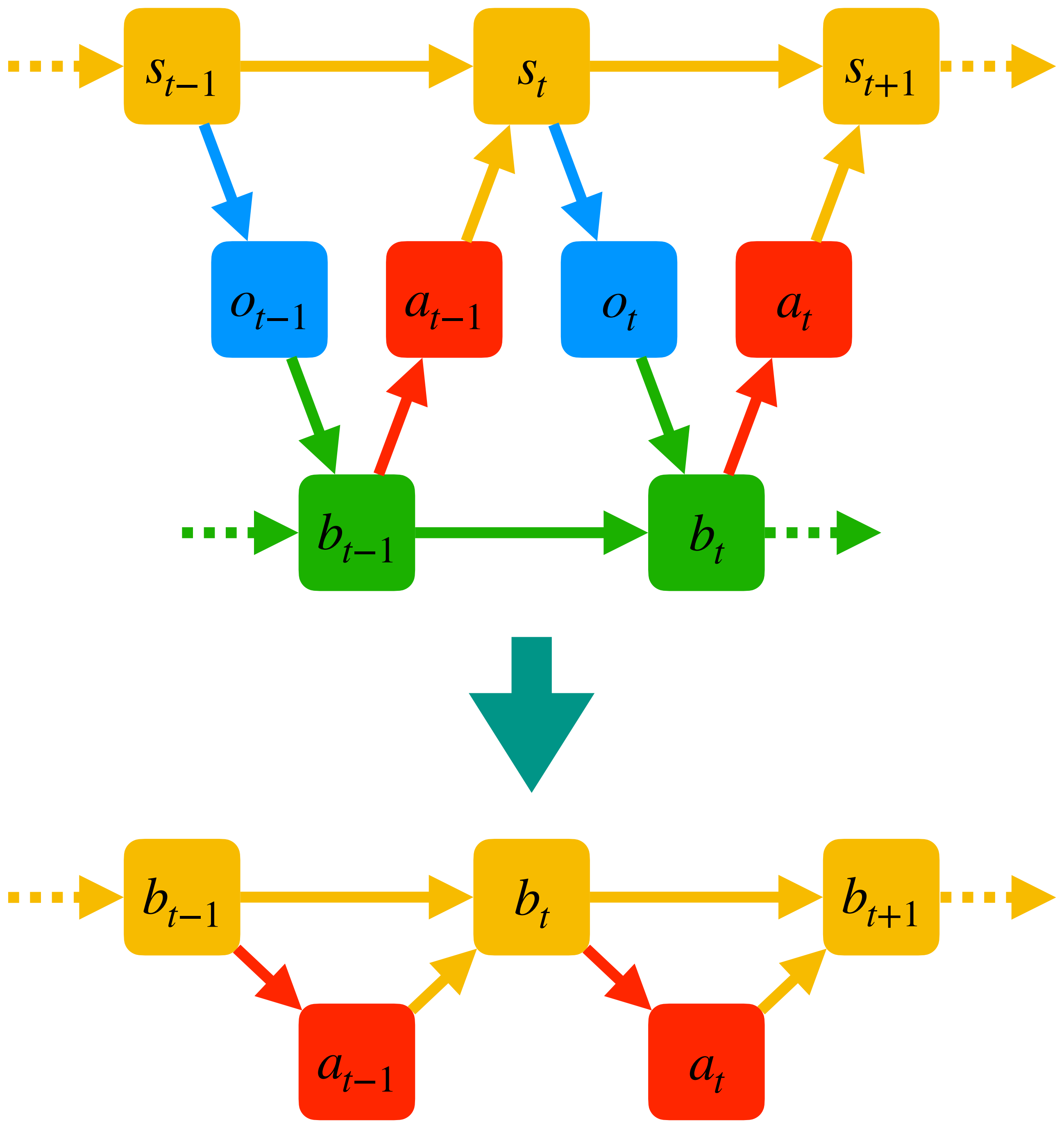
Bayes' rule on o_t · $o_t - s_t - (h_{t-1}, a_{t-1})$ · previous prediction · observation model · normalizer

- A deterministic, model-based update:
  - $b_{t-1}(s_{t-1}) \to$ use a_{t-1} to predict $b'_{t-1}(s_t) \to$ use o_t to update $b_t(s_t)$



Belief-state MDP

- In the linear–quadratic–Gaussian case: certainty equivalence
  - Plan using $\hat{x}_t$ as if it was x_t
- More generally (though vastly less useful): belief-state MDP
  - States: $\Delta(\mathcal{S})$ Actions: $\mathcal{A}$ Rewards: $r(b_t, a_t) = \sum_{s_t} b_t(s_t) r(s_t, a_t)$



- Transitions: each possible observation o_{t+1} contributes its probability

$$p(o_{t+1} \mid b_t, a_t) = \sum_{s_t, s_{t+1}} b_t(s_t) p(s_{t+1} \mid s_t, a_t) p(o_{t+1} \mid s_{t+1})$$

to the total probability that the belief that follows (b_t, a_t, o_{t+1}) is the Bayesian belief

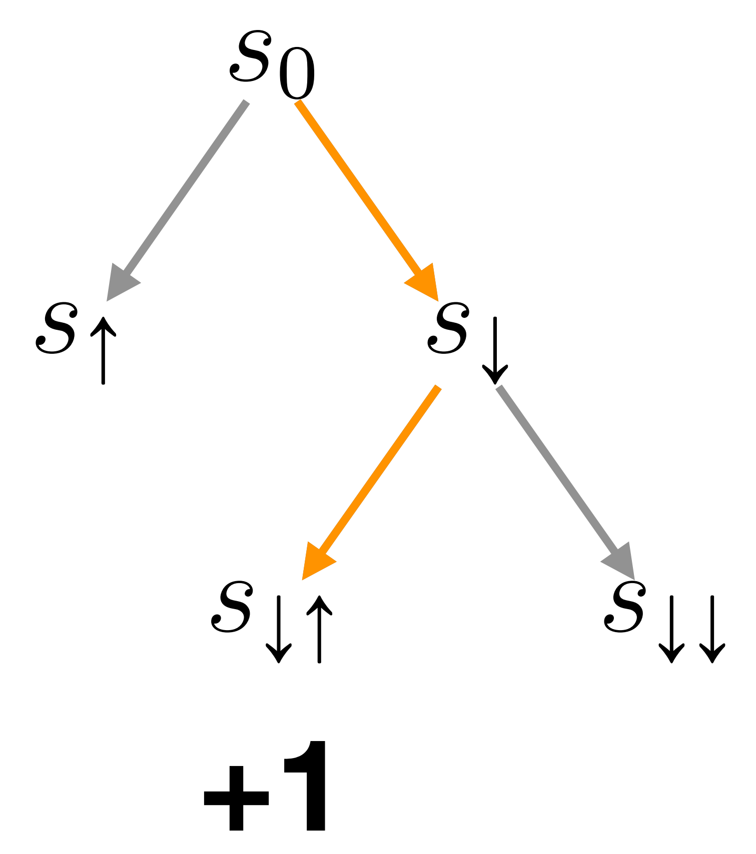
$$b_{t+1}(s_{t+1}) = p(s_{t+1} \mid b_t, a_t, o_{t+1}) = \frac{\sum_{s_t} b_t(s_t) p(s_{t+1} \mid s_t, a_t) p(o_{t+1} \mid s_{t+1})}{p(o_{t+1} \mid b_t, a_t)}$$

Learning to use memory is hard

- Belief space $b(s_t)$ is continuous and high-dimensional (dimension $|\mathcal{S}|$)
  - Curse of dimensionality
  - Beliefs are naturally multi-modal — how do we even represent them?
- The number of reachable beliefs may grow exponentially in t (one per h_t)
  - Curse of history
- Belief-value function can be very complex, hard to approximate
- There may not be optimal stationary deterministic policy ⇒ instability

Stationary deterministic policy counterexample

- Assume no observability
- Stationary deterministic policies gets no reward
- Non-stationary policy: ↓, ↑; expected return: +1
  - But non-stationary = observability of a clock t



- Stationary stochastic policy: ↓ / ↑ with equal prob.; expected return: +0.25
- Open problem: Bellman optimality is inherently stationary and deterministic

no dependence on t → $V(s)$; **maximum achieved for some action** → $\max_a$

$$V(s) = \max_a r(s,a) + \gamma \mathbb{E}_{(s'|s,a)\sim p}[V(s')]$$

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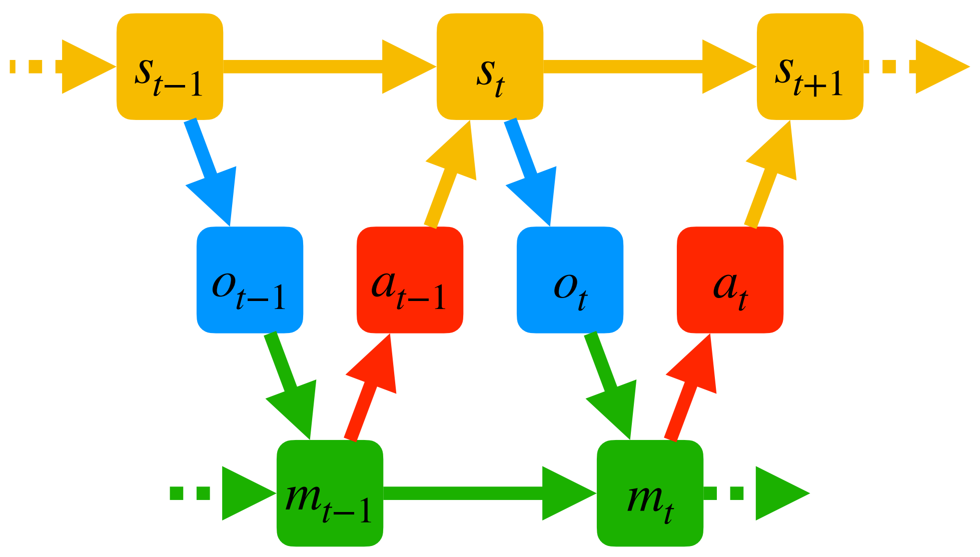
Filtering with function approximation

- Instead of Bayesian belief: memory update $m_t = f_\theta(m_{t-1}, o_t)$ (a_{t-1} optional)
  - Action policy: $\pi_\theta(a_t \,|\, m_t)$
  - Sequential structure = Recurrent Neural Network (RNN)



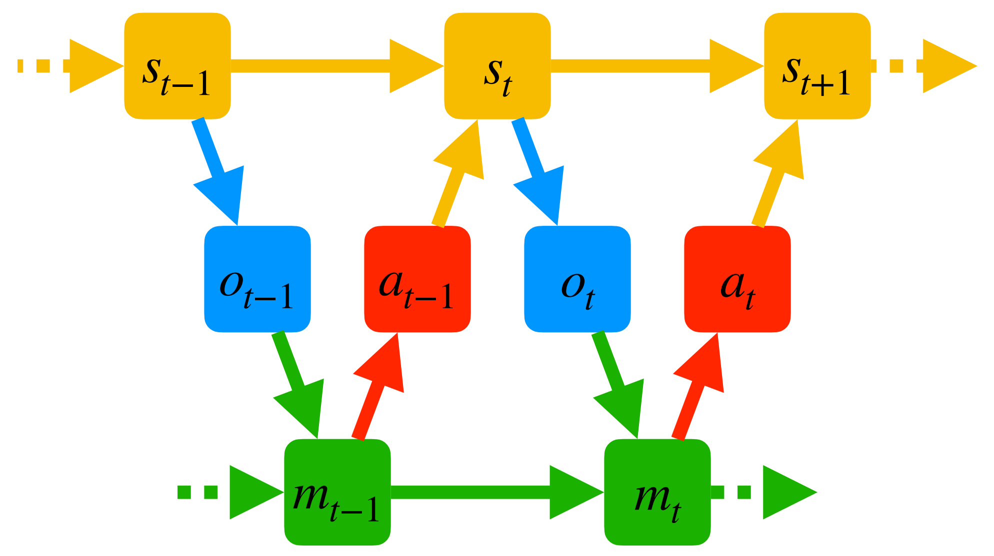
- Training: back-propagate gradients through the whole sequence
  - Back-propagation through time (BPTT)
- Unfortunately, gradients tend to vanish → 0 / explode → ∞
  - Long term coordination of memory updates + actions is challenging
  - RNN can't use information not remembered, but backup no gradient unless used

RNNs in on-policy methods

- Training RNNs with on-policy methods is straightforward (and backward)
  - Roll out policy: parameters of a_t distribution are determined by $\pi_\theta(m_t)$ with

$$m_t = f_\theta(\cdots f_\theta(f_\theta(o_0), o_1), \cdots o_t)$$

  - Compute $\nabla_\theta \log \pi_\theta(a_t | m_t)$ with BPTT all the way to initial observation o_0
- Problems: computation graph > RAM; vanishing / exploding grads
  - Solutions: stop gradients every k steps; use attention
    - Problem: cannot learn longer memory — but that's hard anyway



RNNs in off-policy methods

- Problem: RNN states in replay buffer disagree with current RNN params

- Solution 1: use n-step rollouts to reduce mismatch effect

$$Q_\theta(o_t, m_t, a_t) \to r_t + \gamma r_{t+1} + \cdots + \gamma^{n-1} r_{t+n-1} + \gamma^n \max_{a'} Q_\theta(o_{t+n}, m_{t+n}, a')$$

- Solution 2: "burn in" m_t from even earlier stored steps
  - Same target, but m_t is initialized from $(o_{t-k}, \ldots, o_{t-1})$

- In practice: RNNs not often used, and rarely for long horizons
  - Stacking k frames every step $(o_{t-k+1}, \ldots, o_t)$ may help with short-term memory

Deep RL as partial observability

- Memory-based policies fail us in Deep RL, where we need them most:
  - Deep RL is inherently partially observable
- Consider what deeper layers get as input:
  - High-level / action-relevant state features are not Markov!
- Memory management is a huge open problem in Deep RL
  - Actually, in other areas of ML too: NLP, time-series analysis, video processing, ...

Recap and further considerations

- Let policies depend on observable history through memory
- Memory update: Bayesian, approximate, or learned
  - Learning to update memory is one of the biggest open problems in all of ML
- Let policy be stochastic
  - Should memory be stochastic? interesting research question...
- Let policies be non-stationary if possible, otherwise learning may be unstable
  - Time-dependent policies for finite-horizon tasks
  - Periodic policies for periodic tasks