Toward Provably Unbiased TD Value Estimation

Roy Fox

Department of Computer Science, University of California, Irvine

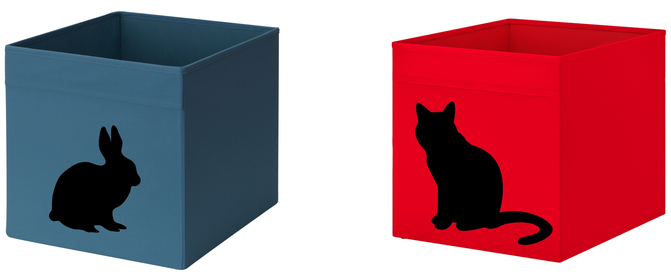
UCI University of California, Irvine

Summary

- We show that the entropy-regularized Bellman operator is **unbiased** when applied to Gaussian Q estimator with inverse-temperature $\beta = \frac{2|\mu_A|}{\sigma_A^2}$
- This β schedule yields a **provably unbiased TD learning** algorithm
- Empirically, such bias reduction **improves convergence**

Why unbiased?

- Bias is like unknown boxes
- Which content is heavier?
- Better scales can't help



Crucial for downstream decisions:

- Exploration
- Further TD updates

Unbiased softmax

Proposition. *Let $a \in \{0,1\}$, and $Q(a)$ a value estimator such that $Q(0)-Q(1) \sim \mathcal{N}(\mu_A, \sigma_A^2)$. For $\beta = \frac{2|\mu_A|}{\sigma_A^2}$, the softmax expectation of Q is an unbiased estimator of the optimal value $\max_a \mathbb{E}[Q(a)]$.*

Entropy-regularized Bellman operator

Regularized Bellman operator: $\mathcal{B}^{\mathcal{F}}[Q](s,a) = \mathbb{E}[r|s,a] + \gamma\, \mathbb{E}_{s'|s,a\sim p}[\mathbb{E}_{a'|s'\sim \pi_Q^{\mathcal{F}}}[Q(s',a')]]$

for $\pi_Q^{\mathcal{F}}(\cdot|s) = \underset{\pi(\cdot|s)}{\operatorname{argmax}}\, \mathbb{E}_{a|s\sim\pi}[\mathcal{F}_{Q,\pi}(s,a)]$

Entropy-regularized objective: $\mathcal{F}_{Q,\pi}(s,a) = Q(s,a) - \beta^{-1} \log \pi(a|s)$

Entropy-regularized Q-learning

Entropy-regularized Q-learning (EQL)

Initialize $Q_1, \ldots, Q_k \sim \mathcal{N}(Q^*, \sigma^2)$

for each (s,a,r,s') **do**

$(\mu_A, \sigma_A^2) \leftarrow (\text{mean}, \text{var})\{Q_i(s',0) - Q_i(s',1)\}_{i=1}^k$

$\beta \leftarrow \frac{2|\mu_A|}{\sigma_A^2}$

$Q_i(s,a) \leftarrow (1-\alpha)Q_i(s,a) + \alpha(r + \text{softmax}_{a'}(Q_i(s',a');\beta)) \quad \forall i = 1,\ldots,k$

Experiments